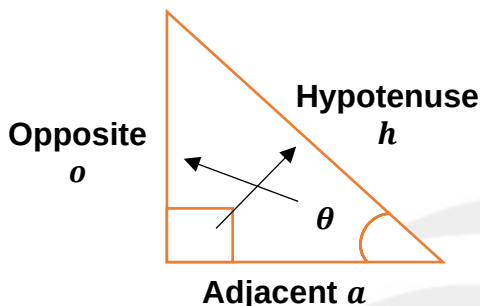


Unit Circle

RECALL: Sine, Cosine & Tangent Ratios



SOH $\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$

CAH $\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$

TOA $\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$

The Unit Circle

The unit circle is a circle with a radius of 1. As you already know, circles are made up of 360° . In radians however, this translates to 2π .

We can solve problems using this and the trigonometric ratios.

Let's discuss the unit circle in relation to $\sin\theta$, $\cos\theta$ and $\tan\theta$.

Notice the angle in the unit circle to the right (its size does not matter).

→ We know that the **hypotenuse = 1** in the triangle as the circle has a radius of 1.

→ We know that x is a length on the x -axis and y is a length on the y -axis

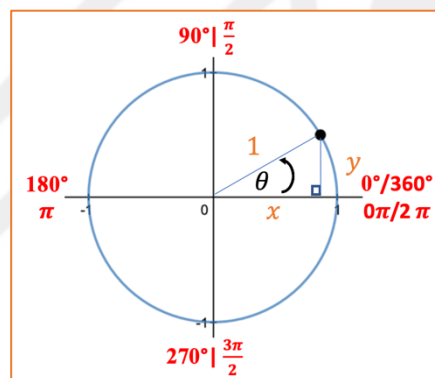
→ We also know **SOH CAH TOA**

We can use this information to further define $\sin\theta$, $\cos\theta$ and $\tan\theta$.

$$x = \cos\theta \quad \text{as} \quad \frac{\text{adj}}{\text{hyp}} = \frac{x}{1} = x$$

$$y = \sin\theta \quad \text{as} \quad \frac{\text{opp}}{\text{hyp}} = \frac{y}{1} = y$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \quad \text{as} \quad \frac{\text{opp}}{\text{adj}} = \frac{y}{x}$$



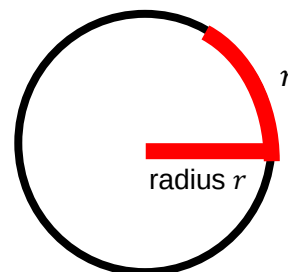
These results state that values on the x -axis relate to $\cos\theta$, values on the y -axis relate to $\sin\theta$ and $\tan\theta$ is a ratio of $\sin\theta$ to $\cos\theta$. This means that it is a ratio of y -values to x -values for a given point in the unit circle.



Understanding Radians

Before we go any further, it is important to understand what exactly radians are. Being able to use conversions between degrees and radians will later become essential.

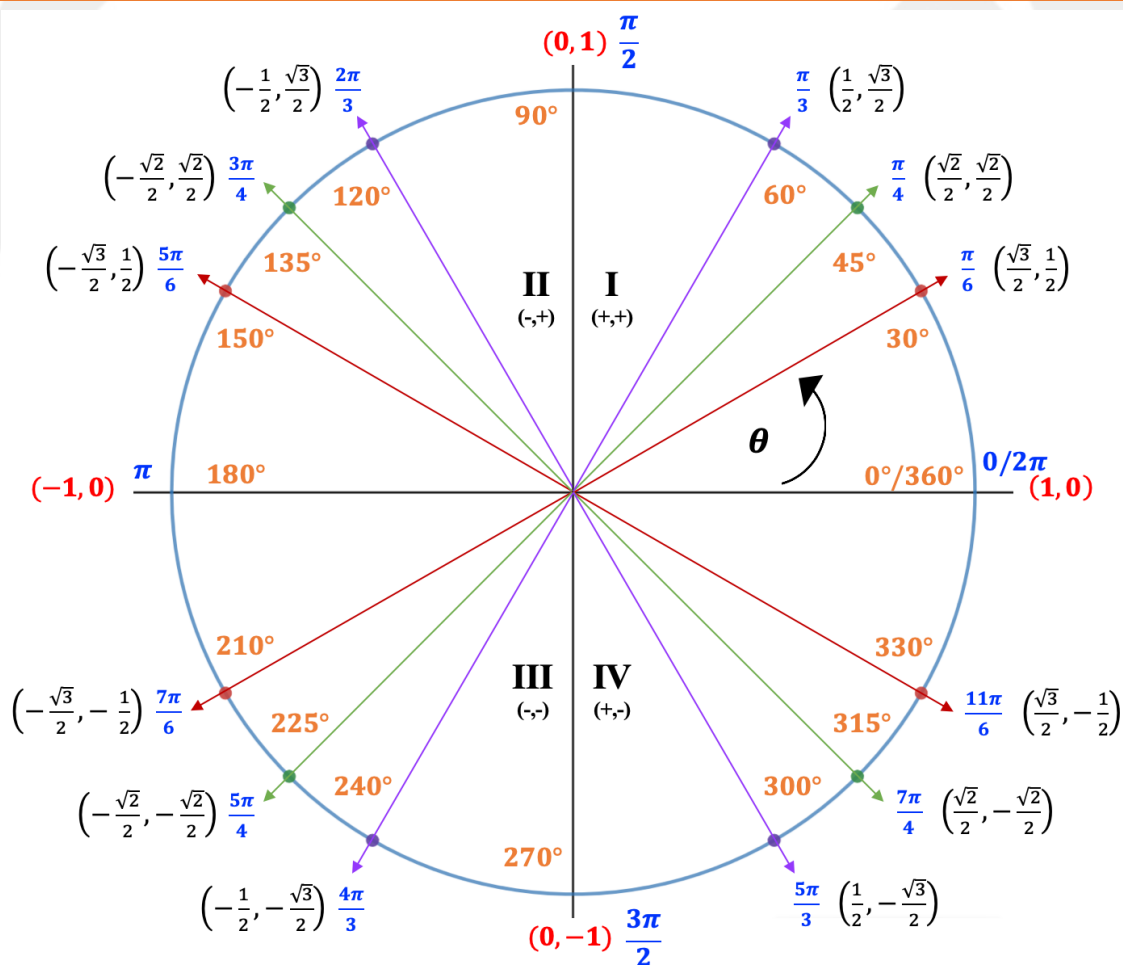
A **radian** is a unit of measurement equal to about 57.3° . It is also the length of the radius around the circle.



For every circle, the circumference equals **2π radians** ($\pi \cong 3.14$). This is equivalent to 360° . If we take half of the circle, which is 180° , we should recognise that it equals to π .

Degrees and radians are just two ways of describing points on a circle.

The Unit Circle - Continued





The Unit Circle – Continued

Identify, discuss and understand these features on the previous page:

Unit circle

- Observe a radius of 1

Direction of angle (θ)

- Angles measured from the line of origin
- Positive angles – measured anti-clockwise
- Negative angles – measured clockwise

Four quadrants (I-IV)

- Coordinate axes divide the Cartesian plane into four quadrants, each are 90° or $(\pi/2)$
- Coordinates can be either positive or negative depending on the quadrant

Sections of pi (π) and (x, y)

- Recognise divisions into thirds, quarters and sixths and their equivalent angles in degrees
- Understand that the coordinates of these sections describe x -values and y -values

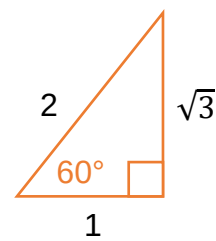
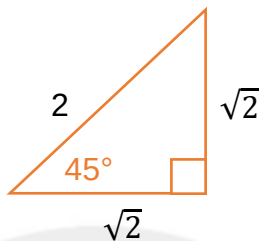
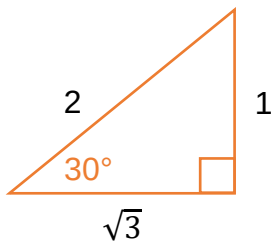
EXAMPLE:

$\frac{\pi}{6}$ has the $(\cos\theta, \sin\theta)$ coordinates $(\frac{\sqrt{3}}{2}, \frac{1}{2})$.

- To learn and remember the coordinate values for each point on the unit circle, refer to the Trigonometric Functions section on the next page

Trigonometric Functions: Degrees & Radians

The three triangles below will help you to memorise the trig functions.



Degrees	Radians	Value
$\cos 0^\circ$ $\cos 360^\circ$	$\cos 0\pi$ $\cos 2\pi$	1
$\sin 0^\circ$ $\sin 360^\circ$	$\sin 0\pi$ $\sin 2\pi$	0
$\cos 90^\circ$	$\cos \frac{\pi}{2}$	0
$\sin 90^\circ$	$\sin \frac{\pi}{2}$	1
$\cos 180^\circ$	$\cos \pi$	-1
$\sin 180^\circ$	$\sin \pi$	0
$\cos 270^\circ$	$\cos \frac{3\pi}{2}$	0
$\sin 270^\circ$	$\sin \frac{3\pi}{2}$	-1

	30°	45°	60°
sin	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

Understand how these values are obtained using the three triangles.

NOTE: $\tan \theta$ does not have any values on the coordinate axes (0° , 90° , 180° and 360°) as it measures tangents that only exist between the axes.

This is because $\tan \theta$ for points on any of the axes contain zeros.
E.g. Finding $\tan \theta$ of $(0, 1)$:
 $\tan 90^\circ = 1/0 = \text{undefined}$

EXAMPLE: Evaluate $\sin 270^\circ$

$$\sin 270^\circ = \sin \left(\frac{3\pi}{2} \right) = -1$$



Instructions: Evaluate the trig functions below.

1	$\sin \frac{\pi}{2}$	10	$\sin \frac{\pi}{3}$	19	$\sin \frac{2\pi}{3}$
2	$\cos \pi$	11	$\sin \frac{\pi}{4}$	20	$\cos \frac{\pi}{3}$
3	$\cos 2\pi$	12	$\cos \frac{5\pi}{6}$	21	$\cos \frac{7\pi}{6}$
4	$\sin 2\pi$	13	$\sin \frac{\pi}{6}$	22	$\sin \frac{11\pi}{6}$
5	$\cos \frac{\pi}{2}$	14	$\cos \frac{2\pi}{3}$	23	$\cos \frac{7\pi}{4}$
6	$\cos \frac{\pi}{6}$	15	$\cos \frac{5\pi}{4}$	24	$\sin \frac{5\pi}{6}$
7	$\sin \frac{3\pi}{2}$	16	$\cos \frac{\pi}{4}$	25	$\cos \frac{5\pi}{3}$
8	$\sin \pi$	17	$\cos \frac{3\pi}{4}$	26	$\sin \frac{7\pi}{6}$
9	$\cos \frac{3\pi}{2}$	18	$\sin \frac{4\pi}{3}$	27	$\sin \frac{3\pi}{4}$



28	$\cos \frac{4\pi}{3}$	37	$\tan \frac{3\pi}{2}$	46	$\tan \frac{5\pi}{3}$
29	$\sin \frac{5\pi}{4}$	38	$\tan \frac{\pi}{3}$	47	$\tan \frac{7\pi}{6}$
30	$\sin \frac{7\pi}{4}$	39	$\tan \frac{5\pi}{6}$	48	$\tan \frac{4\pi}{3}$
31	$\cos \frac{11\pi}{6}$	40	$\tan \frac{3\pi}{4}$	49	$\tan \frac{7\pi}{4}$
32	$\sin \frac{5\pi}{3}$	41	$\tan \pi$	50	$\sin 180^\circ$
33	$\cos \frac{5\pi}{4}$	42	$\tan \frac{5\pi}{4}$	51	$\cos 270^\circ$
34	$\tan \frac{\pi}{2}$	43	$\tan \frac{2\pi}{3}$	52	$\cos 360^\circ$
35	$\tan \frac{\pi}{6}$	44	$\tan \frac{11\pi}{6}$	53	$\cos 0^\circ$
36	$\tan \frac{\pi}{4}$	45	$\tan 2\pi$	54	$\sin 270^\circ$



55	$\cos 180^\circ$	64	$\sin 2\pi + \cos \pi$	73	$\cos 2\pi - \cos \frac{5\pi}{3}$
56	$\sin 120^\circ$	65	$\cos \frac{\pi}{6} - \sin \frac{2\pi}{3}$	74	$\sin \frac{7\pi}{4} + \cos \frac{\pi}{4}$
57	$\cos 45^\circ$	66	$\tan \frac{3\pi}{2} + \sin \frac{\pi}{2}$	75	$\cos \frac{4\pi}{3} - \sin \frac{3\pi}{2}$
58	$\sin 210^\circ$	67	$\cos \frac{2\pi}{3} - \cos \frac{4\pi}{3}$	76	$\sin \frac{11\pi}{6} + \cos 2\pi$
59	$\sin 300^\circ$	68	$\sin \frac{\pi}{4} - \sin \frac{5\pi}{4}$	77	$\cos \frac{5\pi}{3} - \tan \frac{5\pi}{4}$
60	$\cos 315^\circ$	69	$\cos \frac{3\pi}{4} - \cos \frac{7\pi}{4}$	78	$\tan \frac{5\pi}{6} + \tan \frac{7\pi}{6}$
61	$\tan 225^\circ$	70	$\sin \frac{\pi}{6} - \sin \frac{\pi}{2}$	79	$\tan \frac{3\pi}{4} - \tan \frac{7\pi}{4}$
62	$\tan 330^\circ$	71	$\tan \frac{5\pi}{3} + \tan \frac{2\pi}{3}$	80	$\sin 2\pi - \tan \frac{11\pi}{6}$
63	$\tan 135^\circ$	72	$\tan \frac{3\pi}{2} + \sin \frac{4\pi}{3}$	81	$\tan \frac{5\pi}{3} + \tan \frac{2\pi}{3}$