



Logarithms

What are Logarithms?

Logarithms are mathematical operations that tell you how many times a number (known as the base number) should be multiplied by itself to become another number. For example, $\log_2 8 = 3$ ($2 \times 2 \times 2 = 8$).

Logarithms can be thought of as exponents or orders written in a line.

Log Laws

The laws of logarithms are useful in evaluating log expressions.

(1) Addition $\leftrightarrow$ Multiplication

Adding logs with the same base can be simplified through multiplication

$$\log_x A + \log_x B = \log_x (A \times B)$$

(2) Subtraction $\leftrightarrow$ Division

Subtracting logs with the same base can be simplified through division

$$\log_x A - \log_x B = \log_x \left(\frac{A}{B} \right) = \frac{\log_x A}{\log_x B}$$

(3) Index $\leftrightarrow$ Multiplication

The index of a log can be brought down and be written as multiplication

$$\log_x (A^y) = y \times \log_x A$$

(4) Log of 1

Log of 1 (regardless of base) equals 0

$$\log_x (1) = 0$$

(5) Log of base

$$\log_x x = 1$$

(6) Identity

$$\log_x (x^y) = y$$

(7) Identity

$$x^{\log_x (y)} = y$$

(8) Natural log $\log_e$

Log to base e converted to natural log (ln)

$$\begin{aligned} \log_e A &= \ln A \\ \log_e e &= \ln e = 1 \end{aligned}$$



The Natural Logarithm – “ $\log_e$ ” or “ $\ln$ ”

The natural logarithm is a log to the base e , where e is an irrational constant with a value of approximately 2.718 (2.7182818...).

We use natural logs to find the inverse of indices with a base of e .

EXAMPLES

EXAMPLE 1: $\log_7 49$

$$\begin{aligned}\log_7 49 &= \log_7 7^2 \\ &= 2 \times \log_7 7 \\ &= 2 \times 1 \\ &= 2\end{aligned}$$

EXAMPLE 2: $\log_2 3 + \log_2 5$

$$\begin{aligned}\log_2 3 + \log_2 5 &= \log_2 (3 \times 5) \\ &= \log_2 15\end{aligned}$$

Instructions: Evaluate the expressions below using the log laws.

1	$\log_6 6$	5	$\log_3 6 + \log_3 4$
2	$\log_4 6 - \log_4 2$	6	$\log_{10} 1$
3	$\log_4 4$	7	$\log_e e^3$
4	$\log_2 2^3$	8	$8\log_5 5^2$



9	$\log_5 49 - \log_5 7$	15	$\log_3 9$
10	$-4\log_5 5^7$	16	$\log_2 8$
11	$3\log_3 2$	17	$\frac{1}{2}\log_4 16$
12	$\log_3 7 + \log_3 6$	18	$\log_7 49$
13	$9\log_e e^7$	19	$5\log_6 36$
14	$11 \times 7^{\log_7 9}$	20	$\frac{1}{3}\log_2 8$



21	$\log_9 30 - \log_9 5$	27	$\log_4 256 (\log_7 144 - \log_7 16)$
22	$\log_5 125$	28	$-3 \log_5 25 \times \frac{1}{2} \log_8 169$
23	$\log_3 64 - \log_3 16$	29	$\log_2 9 + \log_2 8 - \log_2 6$
24	$\log_2 56 - \log_2 7$	30	$\log_4 64 + \log_6 216$
25	$\log_2 16 + \log_3 27$	31	$\frac{1}{3} \log_3 27 - 3 \log_7 49$
26	$13^{\log_{13} 5} \times \log_2 8$	32	$\frac{1}{2} \log_3 81 - \frac{1}{3} \log_7 343$



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Instructions: Evaluate the expressions below using the log laws.

1	$-18\log_8 8$	5	$2\log_4 4 - \log_4 256$
2	$4^{\log_4 6} \times -4$	6	$\log_6 7 + \log_6 5$
3	$\log_5 42 - \log_5 6$	7	$\log_8 64 + \log_2 32$
4	$-\log_5 125 + 4$	8	$6\log_7 49 - \log_5 125$



9	$\log_4 32 - \log_4 8$	15	$3\log_7 49 + 2\log_6 36$
10	$-2\log_3 81 - 4\log_2 8$	16	$\log_6 216 - \log_7 49$
11	$\log_8 64 + \log_{10} 1000$	17	$\log_5 13 + \log_5 3$
12	$15 - \log_2 64$	18	$\log_7 12 + \log_7 2 - \log_7 6$
13	$\log_2 128$	19	$\log_3 81 - \log_2 256$
14	$\log_5 625$	20	$\log_5 125 \times 6^{\log_6 7}$



21	$-4\log_6 216 + 15$	27	$-\log_{10} 10,000 + \log_3 243$
22	$\log_9 729 + \log_4 64$	28	$\log_8 84 - \log_8 14$
23	$\log_{10} 10,000$	29	$-8\log_2 512$
24	$\log_3 130 - \log_3 5$	30	$7^{\log_7 16} \div 2\log_e e^2$
25	$\log_4 1024 + \log_2 32$	31	$\log_5 625 - \log_7 343 + \log_8 512$
26	$66\log_3 9 \div 3^{\log_3 4}$	32	$\frac{5^{\log_5 12} \times \frac{1}{3} \log_7 343}{2}$