



Polynomial Remainder Theorem

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The polynomial remainder theorem is a concept that allows us to find the remainder of a polynomial long division, without doing long division.

Recall the **dividend**, **divisor** and **quotient** $150 \div 2 = 75$

The remainder theorem states that:

- 1) If a polynomial $f(x)$ is divided by $x - a$ (where a is a constant), the remainder will be $f(a)$.

$$\frac{f(x)}{x - a} = f(a)$$

- 2) The dividend equals the product of the quotient and divisor, plus the remainder. Note: $q(x)$ is a polynomial (one degree less than $f(x)$)

$$f(x) = q(x) \times (x - a) + f(a)$$

EXAMPLE: Prove the polynomial remainder theorem by dividing $f(x) = 3x^2 - 4x + 7$ by $x - 1$

1. The polynomial $f(x)$ divided by $x - 1$:

$$\begin{array}{r} 3x - 1 \\ x - 1 \overline{) 3x^2 - 4x + 7} \\ \underline{3x^2 - 3x} \downarrow \\ 0 - x + 7 \\ \underline{- (-x + 1)} \\ 6 \end{array}$$

2. The remainder of this quotient is 6.

3. If the polynomial remainder theorem is correct, then the remainder of $\frac{f(x)}{x-a}$ should be $f(a)$. Refer to Rule 1.

Since $x - a$ is $x - 1$, $a = 1$.

$$\begin{aligned} f(x) &= 3x^2 - 4x + 7 \\ f(1) &= 3(1)^2 - 4(1) + 7 \\ &= 3 - 4 + 7 \\ &= 6 \end{aligned}$$

Therefore, the polynomial remainder theorem is correct.



Instructions: Answer the questions below.

1	2
Find the remainder if polynomial $f(x)$ is divided by $x - 1$. $f(x) = 12x^2 - 5x - 3$	Find the remainder if polynomial $f(x)$ is divided by $x + 4$. $f(x) = 8x^2 + x - 6$
3	4
Find the remainder if polynomial $f(x)$ is divided by $x - 2$. $f(x) = x^3 - 4x^2 - 7x + 10$	Find the remainder if polynomial $f(x)$ is divided by $x - 3$. $f(x) = 5x^3 + 2x^2 + 6x - 2$



5

Find the quotient if polynomial $f(x) = 3x^2 - 5x - 1$ divided by $x - 3$ gives a remainder of 11.

6

Find the quotient if polynomial $f(x) = 4x^2 - 3x + 4$ divided by $x + 2$ gives a remainder of 26.